The cusp solitons in optical fibres

1996 J. Phys. A: Math. Gen. 29 L141

(http://iopscience.iop.org/0305-4470/29/6/001)

J. Phys. A: Math. Gen. **29** (1996) L141–L142. Printed in the UK

LETTER TO THE EDITOR

The cusp solitons in optical fibres

Li Cheng

Department of Applied Physics, University of Electronic Science and Technology of China, Chengdu 610054, People's Republic of China

Received 14 November 1995

Abstract. We solve the higher-order nonlinear Schrödinger equation, which describes the propagation of femtosecond pulses in nonlinear fibres. A set of new soliton solutions is obtained.

For many years, much attention has been paid to soliton communication in optical fibres owing to its potential applications [1–4]. It is well known that the propagation of picosecond pulses in optical fibres is described by the nonlinear Schrödinger equation (NLS) [1, 2]. However, for ultrashort femtosecond pulses, the NLS is invalid and is substituted with a higher-order nonlinear Schrödinger equation given by [3, 4, 6]

$$\mathrm{i}u_z - \tfrac{1}{2}\alpha u_{tt} + |u|^2 u + \mathrm{i}\epsilon u_{ttt} + \mathrm{i}\delta |u|^2 u_t + \mathrm{i}\rho u^2 u_t^* = 0 \tag{1}$$

where $u = (n_2\omega_0/c)^{1/2}q$, $\alpha = \pm\beta_{\omega\omega}$, $\epsilon = \pm\frac{1}{6}\beta_{\omega\omega\omega}$, $\delta = (4/\omega_0) + (2\gamma/b) > 0$, $\rho = (2/\omega_0) + (2\gamma/b) > 0$, q is the slowly-varying envelope of the electromagnetic field, c is light speed, n_2 is the nonlinear index of refraction, $\beta_{\omega\omega}$ and $\beta_{\omega\omega\omega}$ are the dispersion parameters evaluated at the carrier frequency ω_0, b is the radius of the frequency-dependent electromagnetic mode propagation in the fibres, γ is a parameter depending on the fibre geometry and z and t represent the space and time coordinates, respectively. If $\rho = 0$, equation (1) becomes the Hirota equation [5], of which there exist N-soliton solutions at $\epsilon = -\frac{1}{6}\alpha\delta$. For $\rho \neq 0$, equation (1) is no longer integrable. However, the hyperbolic secant (bright) and hyperbolic tangent (dark) soliton solutions for equation (1) have been obtained [6]. In this letter, we present a set of cusp solitary wave solutions of equation (1). When the cusps are rounded off, we recover these solutions in [6].

We assume

$$u = \mathrm{e}^{\mathrm{i}(Rt+\Omega z+\Phi_0)}W(z,t) \tag{2}$$

where R, Ω and Φ_0 are real constants and W is the amplitude. Substituting equation (2) into equation (1), we have

$$\begin{cases} W_{tt} = AW + BW^3 \\ W_z + (\epsilon A - \alpha R - 3\epsilon R^2)W_t + (3\epsilon B + \delta + \rho)W^2 W_t = 0 \end{cases} \tag{3}$$

where, A and B are real constants and

$$R = \frac{\frac{1}{2}\alpha B - 1}{\rho - \delta - 3\epsilon B} \qquad \Omega = \tfrac{1}{2}\alpha R^2 + \epsilon R^3 - A(\tfrac{1}{2}\alpha + 3\epsilon R). \tag{4}$$

For equations (3), we obtain the exact solutions when A is a parameter and $B = -(\delta+\rho)/3\epsilon$. Let $\xi = t + (\alpha R + 3\epsilon R^2 - \epsilon A)z$, then equations (3) can be rewritten as

$$W_{\xi\xi} = AW + BW^3. \tag{5}$$

0305-4470/96/060141+02$19.50

It is easy to find the first integral of equation (5)

$$W_\xi = \pm(AW^2 + \tfrac{1}{2}BW^4 - C)^{1/2} \tag{6}$$

where the integral constant $C = (AW^2 + \frac{1}{2}BW^4)|_{\xi=\pm\infty} = 0$ or $-A^2/2B$.

From equation (6), we have following solutions.

(1) When $A > 0$, $B > 0$, (i.e. $\epsilon < 0$),

$$W = \frac{\sqrt{8AP/B}}{P\mathrm{e}^{\sqrt{A}|\xi-\xi_0|} - \mathrm{e}^{-\sqrt{A}|\xi-\xi_0|}} \qquad P = \text{constant} > 1. \tag{7}$$

(2) When $A > 0$, $B < 0$ (i.e. $\epsilon > 0$),

$$W = \frac{\sqrt{-8AP/B}}{P\mathrm{e}^{\sqrt{A}|\xi-\xi_0|} + \mathrm{e}^{-\sqrt{A}|\xi-\xi_0|}} \qquad P = \text{constant} > 0. \tag{8}$$

(3) When $A < 0$, $B > 0$,

$$W = \sqrt{-\frac{A}{B}}\,\frac{P\mathrm{e}^{\sqrt{-A/2}|\xi-\xi_0|} + \mathrm{e}^{-\sqrt{-A/2}|\xi-\xi_0|}}{P\mathrm{e}^{\sqrt{-A/2}|\xi-\xi_0|} - \mathrm{e}^{-\sqrt{-A/2}|\xi-\xi_0|}} \qquad P = \text{constant} > 1 \tag{9}$$

and

$$W = \sqrt{-\frac{A}{B}}\,\frac{P\mathrm{e}^{\sqrt{-A/2}|\xi-\xi_0|} - \mathrm{e}^{-\sqrt{-A/2}|\xi-\xi_0|}}{P\mathrm{e}^{\sqrt{-A/2}|\xi-\xi_0|} + \mathrm{e}^{-\sqrt{-A/2}|\xi-\xi_0|}} \qquad P = \text{constant} \geqslant 1 \tag{10}$$

where ξ_0 is an arbitrary real constant.

The solutions (7)–(10) are of the solitary wave shape with a cusp at ξ_0. Obviously, equations (8) and (10) are nothing but the hyperbolic secant and hyperbolic tangent soliton solutions, respectively, presented previously, if the cusps are rounded off (i.e. $P = 1$) [6].

In summary, we have obtained a set of cusp soliton solutions of the higher-order nonlinear Schrödinger equation (1). These results are expected to have an important application in future optical communications and other research.

References

[1] Hasegawa A and Tappert F 1973 *Appl. Phys. Lett.* **23** 171
[2] Mollenauer L F, Stolen R H and Gordon J P 1980 *Phys. Rev. Lett.* **45** 1095
[3] Kodama Y 1985 *J. Stat. Phys.* **39** 597
[4] Potasek M J 1989 *J. Appl. Phys.* **65** 941
[5] Hirota R 1973 *J. Math. Phys.* **14** 805
[6] Potasek M J and Tabor M 1991 *Phys. Lett.* **154A** 449